REMARKS/ARGUMENTS

Favorable reconsideration of this application as presently amended and in light of the following discussion is respectfully requested.

Claims 1-24 are pending in the present application. Claims 1-5, 8-13, and 16-21 are amended by the present amendment.

In the outstanding Office Action, the title was objected to; Claims 3, 5, 10, 12, 18, and 20 were objected to; Claims 1-24 were rejected under 35 U.S.C. § 112, second paragraph; Claims 1, 2, 7-9, 14-17, and 22-24 were rejected under 35 U.S.C. § 103(a) as unpatentable over Nagara (U.S. Patent No. 6,731,584) in view of Koike et al. (U.S. Patent No. 5,625,616, herein "Koike"); and Claims 3-6, 10-13, and 18-21 were indicated as allowable if rewritten in independent form.

Applicants thanks the examiner for the indication of allowable subject matter. However, Applicants believe that independent Claims 1, 8, and 16 patentably distinguish over the applied art and thus, Claims 3-6, 10-13, and 18-21 have been maintained in dependent form.

Regarding the objection to the title, the title has been amended to be more indicative of the claimed invention. No new matter has been added. Accordingly, it is respectfully requested this objection be withdrawn.

Regarding the objection to Claims 3, 5, 10, 12, 18, and 20, these claims have been amended as suggested by the outstanding Office Action. No new matter has been added. Accordingly, it is respectfully requested this objection be withdrawn.

Regarding the rejection of Claims 1-24 under 35 U.S.C. § 112, second paragraph, Claims 1, 2, 8, 9, 16, and 17 have been amended to recite a single plurality of amplitudes, Claims 3, 5, 10, 12, 18, and 20 have been amended to recite that a power output level is maintained at a constant level by adjusting a difference of a preset driving current and a

measured driving current, as shown for example in Figures 2 and 5, and Claims 4, 11, and 19 have been amended as suggested by the outstanding Office Action. No new matter has been added. Accordingly, it is respectfully requested this rejection be withdrawn.

The rejection of Claims 1, 2, 7-9, 14-17, and 22-24 under 35 U.S.C. § 103(a) as unpatentable over Nagara in view of Koike is respectfully traversed for the following reasons.

Briefly recapitulating, independent Claim 1 is directed to a laser driving apparatus including, inter alia, a current monitoring unit for monitoring a drive current, an amplitude control unit for controlling an amplitude of a radio frequency current to be superimposed on the drive current, and a control unit for controlling the amplitude of the radio frequency current. The amplitude is controlled based on current values of the drive current obtained by the current monitoring unit at a plurality of the amplitudes of the radio frequency current or based on detection values of an optical output of the laser at the plurality of the amplitudes of the radio frequency current. Independent Claims 8 and 16 recite similar features as Claim 1.

Turning to the applied art, Nagana shows in Figure 1 an optical information reproducing apparatus having a laser 5 that is controlled by a driving unit 21. A DC signal of the driving unit 21 is superposed with an RF signal from an oscillator 23. An amplitude of the RF signal is based only on an erasing power Pe and an average power Pr as discussed next.

Nagana discloses at column 4, lines 8-10, that a controller 16 controls the oscillator 23 to output the high frequency sine-wave signal S1. The same controller 16 "establishes an average power value Pr of laser beam L1" to be less than half of an erasing power Pe as disclosed by Nagana at column 4, lines 29-34. An amplitude AMP of the high frequency signal S1 is "converted into the power of laser beam L1" and is represented by MOD as disclosed in Nagana at column 4, lines 51-54. The amplitude MOD satisfies equation (2) in Nagana (see column 4, line 64). Equation (2) of Nagana limits the amplitude MOD to be

larger than the average power Pr and smaller than twice the difference between the erasing power Pe and the average power Pr.

However, Nagana does not teach or suggest that the controller 16 determines the amplitude MOD based on current values of the drive current **at plural amplitudes** of the radio frequency current as required by independent Claims 1, 8, and 16. In this respect, it is noted that Nagana uses only an average power Pr and an erasing power Pe (a constant) to determine the amplitude MOD, and these values are not determined at **plural amplitudes of the radio frequency current**.

Even if the outstanding Office Action interprets Claims 1, 8, and 16 to require that the amplitude of the radio frequency current is determined based on detection values of the optical output of the laser, this operation is still performed by the claimed device at plural amplitudes of the radio frequency current, a feature that is not disclosed or suggested by Nagana.

The outstanding Office Action relies on Koike for teaching a current monitoring unit for monitoring a drive current. However, Koike does not cure the deficiencies of Nagana discussed above with regard to Claim 1.

Accordingly, it is respectfully submitted that independent Claims 1, 8, and 16 and each of the claims depending therefrom patentably distinguish over Nagana and Koike, either alone or in combination.

Consequently, in light of the above discussion and in view of the present amendment, the present application is believed to be in condition for allowance and an early and favorable action to that effect is respectfully requested.

Respectfully submitted,

OBLON, SPIVAK, McCLELLAND,
MAIER & NEUSTADT, P.C.

R Fetea

Bradley D. Lytle
Attorney of Record
Registration No. 40,073

Remus F. Fetea, Ph.D.
Registration No. 59,140

Customer Number
22850

Tel: (703) 413-3000
Fax: (703) 413 -2220
(OSMMN 06/04)

I:\ATTY\RFF\27S\275835\275835US AM 7-16-07.DOC

Proof: (34) comes from the definition of Σ. (35) can be shown by construction. Let us take

$$\sqrt{\mathsf{SNR}}\mathbf{B} \triangleq \sqrt{c \cdot \min\{\lambda_{\max}^{-1}(\mathbf{H}\mathbf{H}^{\dagger}), 1\}} \cdot \mathbf{I}. \tag{36}$$

Then, the power constraint (6) is always satisfied with $c = \left(\mathsf{SNR}^{-1} + \pi_1 \rho\right)^{-1}$. Since $c \doteq \mathsf{SNR}^{0}$, we have

$$1 \leq 1 + \lambda_{\max}(\mathbf{P}\mathbf{P}^{\dagger}) \,\dot{\leq}\, 1 + \mathsf{SNR}^{-\alpha_{\max}} \doteq \mathsf{SNR}^{0} \tag{37}$$

where $\alpha_{\max}$ is the exponential order of $\lambda_{\max}(\mathbf{G}^{\dagger}\mathbf{G})$ and is positive with probability 1 in the high SNR regime [6], [10]. ■

By lemma 5 and the concavity of log det on positive matrices, we have

$$\log\det\left(\mathbf{I} + \mathsf{SNR}\hat{\mathbf{H}}\hat{\mathbf{H}}^{\dagger}\right) \geq \log\det\left(\mathbf{I} + \mathsf{SNR}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\dagger}\right) \geq \log\det\left(\mathbf{I} + \mathsf{SNR}(1 + \lambda_{\max}(\mathbf{P}\mathbf{P}^{\dagger}))^{-1}\hat{\mathbf{H}}\hat{\mathbf{H}}^{\dagger}\right)$$

with

$$\hat{\mathbf{H}} \triangleq \begin{bmatrix} \mathbf{F} & \mathbf{O} \\ \mathbf{P}\mathbf{H} & \mathbf{F} \end{bmatrix}.$$

Therefore, with $\mathbf{B}$ in (36), we have $\log\det\left(\mathbf{I} + \mathsf{SNR}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\dagger}\right) \doteq \log\det\left(\mathbf{I} + \mathsf{SNR}\hat{\mathbf{H}}\hat{\mathbf{H}}^{\dagger}\right)$. Assume that in the rest of the proof, we always consider $\mathbf{B}$ being in the form (36). Then we have

$$\mathcal{I}_{\max} \triangleq \max_{\mathbf{B}\in\mathcal{B}} \mathcal{I}(\mathbf{x}; \sqrt{\mathsf{SNR}}\tilde{\mathbf{H}}\mathbf{x} + \mathbf{z}) \,\dot{\geq}\, \log\det\left(\mathbf{I} + \mathsf{SNR}\hat{\mathbf{H}}\hat{\mathbf{H}}^{\dagger}\right) \tag{38}$$

where $\mathcal{B}$ is the set of matrices $\mathbf{B}$ that satisfy the power constraint (6). Define $\mathbf{M} \triangleq \mathbf{I} + \mathsf{SNR}\hat{\mathbf{H}}\hat{\mathbf{H}}^{\dagger}$, we have

$$\mathbf{M} \triangleq \begin{bmatrix} \mathbf{I} + \mathsf{SNR}\mathbf{F}\mathbf{F}^{\dagger} & \mathsf{SNR}\mathbf{F}\mathbf{H}^{\dagger}\mathbf{P}^{\dagger} \\ \mathsf{SNR}\mathbf{P}\mathbf{H}\mathbf{F}^{\dagger} & \mathbf{I} + \mathsf{SNR}\left(\mathbf{F}\mathbf{F}^{\dagger} + \mathbf{P}\mathbf{H}\mathbf{H}^{\dagger}\mathbf{P}^{\dagger}\right) \end{bmatrix}.$$

Using the identity

$$\det\left(\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}\right) = \det(\mathbf{A})\det(\mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B})$$

and some basic manipulations, we have

$$\det(\mathbf{M}) = \det(\mathbf{I} + \mathsf{SNR}\mathbf{F}\mathbf{F}^{\dagger})\det\left(\mathbf{I} + \mathsf{SNR}\mathbf{F}\mathbf{F}^{\dagger} + \mathsf{SNR}\mathbf{P}\mathbf{H}\boldsymbol{\Omega}\mathbf{H}^{\dagger}\mathbf{P}^{\dagger}\right) \tag{39}$$

where $\boldsymbol{\Omega} \triangleq \mathbf{I} - \mathsf{SNR}\mathbf{F}^{\dagger}(\mathbf{I} + \mathsf{SNR}\mathbf{F}\mathbf{F}^{\dagger})^{-1}\mathbf{F}$ is positive definite. By the matrix inversion lemma $(\mathbf{I} + \mathbf{L}\mathbf{C}\mathbf{R})^{-1} = \mathbf{I} - \mathbf{L}(\mathbf{R}\mathbf{L} + \mathbf{C}^{-1})^{-1}\mathbf{R}$, we have

$$\boldsymbol{\Omega} = (\mathbf{I} + \mathsf{SNR}\mathbf{F}^{\dagger}\mathbf{F})^{-1}.$$

From (38) and (39), we can obtain two lower bounds on $\mathcal{I}_{\max}$. The first one is

$$\mathcal{I}_{\max} \dot{\geq} 2\log\det(\mathbf{I}+\mathsf{SNR}\mathbf{F}\mathbf{F}^{\dagger}). \tag{40}$$

The second lower bound on $\mathcal{I}_{\max}$ is

$$\begin{aligned}
\mathcal{I}_{\max} &\dot{\geq} \log\det(\mathbf{I}+\mathsf{SNR}\mathbf{F}\mathbf{F}^{\dagger}) + \log\det(\mathbf{I}+\mathsf{SNR}\mathbf{P}\mathbf{H}\Omega\mathbf{H}^{\dagger}\mathbf{P}^{\dagger}) \\
&= \log\det(\mathbf{I}+\mathsf{SNR}\mathbf{F}^{\dagger}\mathbf{F}) + \log\det(\mathbf{I}+\mathsf{SNR}\Omega\mathbf{H}^{\dagger}\mathbf{P}^{\dagger}\mathbf{P}\mathbf{H}) \\
&= \log\det(\mathbf{I}+\mathsf{SNR}\mathbf{F}^{\dagger}\mathbf{F}+\mathsf{SNR}\mathbf{H}^{\dagger}\mathbf{P}^{\dagger}\mathbf{P}\mathbf{H}) \\
&\geq \log\det(\mathbf{I}+\mathsf{SNR}\mathbf{H}^{\dagger}\mathbf{P}^{\dagger}\mathbf{P}\mathbf{H}).
\end{aligned} \tag{41}$$

Since in (36), $\min\left\{\lambda_{\max}^{-1}\left(\mathbf{H}\mathbf{H}^{\dagger}\right),1\right\} \doteq \mathsf{SNR}^{0}$, from (9) and (41), we have

$$\mathcal{I}_{\max} \dot{\geq} \log\det\left(\mathbf{I}+\mathsf{SNR}\mathbf{H}^{\dagger}\mathbf{G}^{\dagger}\mathbf{G}\mathbf{H}\right). \tag{42}$$

The outage probability is

$$\begin{aligned}
\mathrm{Prob}\left\{\mathcal{I}_{\max} < 2r\log\mathsf{SNR}\right\} &\dot{\leq} \mathrm{Prob}\left\{\begin{array}{c} 2\log\det(\mathbf{I}+\mathsf{SNR}\mathbf{F}\mathbf{F}^{\dagger}) \leq 2r\log\mathsf{SNR}, \\ \log\det(\mathbf{I}+\mathsf{SNR}\mathbf{H}^{\dagger}\mathbf{G}^{\dagger}\mathbf{G}\mathbf{H}) \leq 2r\log\mathsf{SNR} \end{array}\right\} \\
&= \mathrm{Prob}\left\{2\log\det(\mathbf{I}+\mathsf{SNR}\mathbf{F}\mathbf{F}^{\dagger}) \leq 2r\log\mathsf{SNR}\right\} \\
&\quad \cdot\mathrm{Prob}\left\{\log\det(\mathbf{I}+\mathsf{SNR}\mathbf{H}^{\dagger}\mathbf{G}^{\dagger}\mathbf{G}\mathbf{H}) \leq 2r\log\mathsf{SNR}\right\} \\
&\doteq \mathsf{SNR}^{-\left(d_{\mathbf{F}}(r)+d_{\mathbf{GH}}(2r)\right)}
\end{aligned}$$

where the second lines follows from the independency between $\mathbf{F}$ and $\mathbf{GH}$.

APPENDIX IV
PROOF OF THEOREM 2 AND COROLLARY 1

A. Proof of Theorem 2

As in the case of the single-relay channel, we need two lower bounds on the mutual information. Since the mutual information of the N-relay channel is the sum of that of the N single-relay channels, these two lower bound can be obtained directly from (40) and (42)

$$\begin{aligned}
\mathcal{I}(\mathbf{x};\sqrt{\mathsf{SNR}}\tilde{\mathbf{H}}\mathbf{x}+\mathbf{z}) &\dot{\geq} 2N\log\det(\mathbf{I}+\mathsf{SNR}\mathbf{F}\mathbf{F}^{\dagger}) \\
\mathcal{I}(\mathbf{x};\sqrt{\mathsf{SNR}}\tilde{\mathbf{H}}\mathbf{x}+\mathbf{z}) &\dot{\geq} \sum_{i=1}^{N}\log\det(\mathbf{I}+\mathsf{SNR}\mathbf{H}_i^{\dagger}\mathbf{G}_i^{\dagger}\mathbf{G}_i\mathbf{H}_i)
\end{aligned}$$

The outage probability is upper bounded by

$$\mathrm{Prob}\left\{\mathcal{I}(\mathbf{x};\sqrt{\mathsf{SNR}}\tilde{\mathbf{H}}\mathbf{x}+\mathbf{z}) < 2Nr\log\mathsf{SNR}\right\}$$
$$\leq \mathrm{Prob}\left\{\begin{array}{c} 2N\log\det(\mathbf{I}+\mathsf{SNR}\mathbf{F}\mathbf{F}^\dagger)\leq 2Nr\log\mathsf{SNR}, \\ \sum_{i=1}^N \log\det(\mathbf{I}+\mathsf{SNR}\mathbf{H}_i^\dagger\mathbf{G}_i^\dagger\mathbf{G}_i\mathbf{H}_i)\leq 2Nr\log\mathsf{SNR}\end{array}\right\} \tag{43}$$

Let us denote $\mathcal{I}_i \triangleq \log\det(\mathbf{I}+\mathsf{SNR}\mathbf{H}_i^\dagger\mathbf{G}_i^\dagger\mathbf{G}_i\mathbf{H}_i)$ and $\boldsymbol{\alpha}_i$ the set of exponential orders of the ordered eigenvalues of $\mathbf{H}_i{}^\dagger\mathbf{G}_i{}^\dagger\mathbf{G}_i\mathbf{H}_i$. The pdf of $\boldsymbol{\alpha}_i$ is $p_{\boldsymbol{\alpha}_i} \doteq \mathsf{SNR}^{-d_{\boldsymbol{\alpha}_i}}$ where from (31), $d_{\boldsymbol{\alpha}_i}$ is nondecreasing with respect to the component-wise inequality, i.e.,

$$d_{\boldsymbol{\alpha}_i'} \geq d_{\boldsymbol{\alpha}_i} \text{ if } \boldsymbol{\alpha}_i' \succeq \boldsymbol{\alpha}_i. \tag{44}$$

Let us define

$$\begin{aligned} \mathcal{O}_g(r) &\triangleq \left\{\{\boldsymbol{\alpha}_i\}_{i=1}^N \quad : \quad \sum_{i=1}^N\sum_{k=1}^q(1-\alpha_{i,k})^+\leq 2Nr\right\} \\ \mathcal{O}_i(r) &\triangleq \left\{\boldsymbol{\alpha}_i \quad : \quad \sum_{k=1}^q(1-\alpha_{i,k})^+\leq r\right\} \end{aligned}$$

Then, the outage probability is

$$\begin{aligned} \mathrm{Prob}\left\{\sum_{i=1}^N \mathcal{I}_i \leq 2Nr\log\mathsf{SNR}\right\} &\doteq \mathrm{Prob}\left\{\{\boldsymbol{\alpha}_i\}_{i=1}^N \in \mathcal{O}_g(r)\right\} \\ &\doteq \mathsf{SNR}^{-d_{\mathrm{relay}}} \end{aligned}$$

where d_{relay} is

$$\begin{aligned} d_{\mathrm{relay}} &= \inf_{\mathcal{O}_g(r)}\sum_{i=1}^N d_{\boldsymbol{\alpha}_i} \\ &= \inf_{\boldsymbol{\theta}:\ \sum_i\theta_i=1}\left(\sum_{i=1}^N \inf_{\mathcal{O}_i(2N\theta_i r)} d_{\boldsymbol{\alpha}_i}\right) \\ &= \inf_{\boldsymbol{\theta}:\ \sum_i\theta_i=1}\left(\sum_{i=1}^N d_{\mathbf{G}_i\mathbf{H}_i}(2N\theta_i r)\right) \end{aligned}$$

with the second equality from the fact that the minimal elements lie always in the boundary when (44) is true. For $\mathbf{G}_i\mathbf{H}_i$ same distributed for all i, we can have (13) by the convexity of $d_{\mathbf{GH}}$.

B. Proof of Corollary 1

For simplicity, we prove the particular case $N = 1$ here. For $N > 1$, same method applies. The lower bound is a direct consequence of theorem 2. The upper bound can be found by relaxing the half duplex constraint, i.e., $\tilde{\mathbf{H}} \triangleq \mathbf{\Gamma}\,[\mathbf{P}\mathbf{H} \quad \mathbf{F}]$ with all matrices being similarly defined as before. Define $\hat{\mathbf{H}} \triangleq [\mathbf{P}\mathbf{H} \quad \mathbf{F}]$. First, since $\mathbf{I} \succeq \mathbf{\Gamma}^\dagger\mathbf{\Gamma}$, we have

$$\begin{aligned}
\mathcal{I}(\mathbf{x}; \sqrt{\mathsf{SNR}}\tilde{\mathbf{H}}\mathbf{x} + \mathbf{z}) &= \log\det\left(\mathbf{I} + \mathsf{SNR}\hat{\mathbf{H}}^\dagger\mathbf{\Gamma}^\dagger\mathbf{\Gamma}\hat{\mathbf{H}}\right) \\
&\leq \log\det\left(\mathbf{I} + \mathsf{SNR}\mathbf{F}\mathbf{F}^\dagger + \mathsf{SNR}\mathbf{P}\mathbf{H}\mathbf{H}^\dagger\mathbf{P}^\dagger\right) && (45)\\
&= \log\det\left(\mathbf{I} + \mathsf{SNR}\mathbf{F}\mathbf{F}^\dagger + \mathsf{SNR}\mathbf{G}(\mathsf{SNR}\mathbf{B}\mathbf{H}\mathbf{H}^\dagger\mathbf{B}^\dagger)\mathbf{G}^\dagger\right) && (46)\\
&\dot{\leq} \log\det\left(\mathbf{I} + \mathsf{SNR}\mathbf{F}\mathbf{F}^\dagger + \mathsf{SNR}\mathbf{G}\mathbf{G}^\dagger\right) && (47)
\end{aligned}$$

where λ_i, μ_i are ordered eigenvalues of $\mathbf{F}\mathbf{F}^\dagger, \mathbf{G}\mathbf{G}^\dagger$, respectively and α_i, β_i are exponential orders of λ_i, μ_i. Therefore, the channel $\tilde{\mathbf{H}}$ is asymptotically worse than the channel $[\mathbf{G} \quad \mathbf{F}]$ in the high SNR regime and we have $d_{\mathbf{F}}(0) + d_{\mathbf{G}}(0) \geq d^{\mathsf{MIMO}}_{\mathsf{NAF},N}(0)$.

Then, since $\mathbf{I} \succeq \mathbf{\Gamma}^\dagger\mathbf{P}^\dagger\mathbf{P}\mathbf{\Gamma}$, another bound is

$$\begin{aligned}
\mathcal{I}(\mathbf{x}; \sqrt{\mathsf{SNR}}\tilde{\mathbf{H}}\mathbf{x} + \mathbf{z}) &\leq \log\det\left(\mathbf{I} + \mathsf{SNR}\mathbf{\Gamma}\mathbf{F}\mathbf{F}^\dagger\mathbf{\Gamma}^\dagger\right) + \log\det\left(\mathbf{I} + \mathsf{SNR}\mathbf{\Gamma}\mathbf{P}\mathbf{H}\mathbf{H}^\dagger\mathbf{P}^\dagger\mathbf{\Gamma}^\dagger\right) && (48)\\
&\leq \log\det\left(\mathbf{I} + \mathsf{SNR}\mathbf{F}^\dagger\mathbf{F}\right) + \log\det\left(\mathbf{I} + \mathsf{SNR}\mathbf{H}^\dagger\mathbf{H}\right) && (49)
\end{aligned}$$

from which we have $d_{\mathbf{F}}(0) + d_{\mathbf{H}}(0) \geq d^{\mathsf{MIMO}}_{\mathsf{NAF},N}(0)$.

When the channel is Rayleigh, proposition 1 applies. As indicated in remark 1, we have $d_{\mathbf{GH}} = \min\{d_{\mathbf{G}}, d_{\mathbf{H}}\}$ for $|m - n| \geq l - 1$.

Appendix V
Proof of Theorem 3

To prove theorem 3, it is enough to show that in the high SNR regime, an error occurs with the rate-n NVD code $\mathcal{X}$ only when the channel is in outage for a rate $\frac{q}{n}r$. To this end, we will show that the error event set of $\mathcal{X}$ is actually included in the outage event set.

A. Outage event

For a channel $\mathbf{H}$, the outage event at high SNR is [10]

$$\mathcal{O}(r) \doteq \left\{\mathbf{H} \;:\; \log\det\left(\mathbf{I} + \mathsf{SNR}\mathbf{H}\mathbf{H}^\dagger\right) < r\log\mathsf{SNR}\right\}.$$

Let us develop the determinant as[8]

$$\det(\mathbf{I}+\mathsf{SNR}\mathbf{H}\mathbf{H}^\dagger) = 1 + \sum_{i=1}^{q} \mathsf{SNR}^i D_i(\mathbf{H}\mathbf{H}^\dagger)$$

where $D_i(\mathbf{M})$ is the sum of $\binom{q}{i}$ products of i different eigenvalues of $\mathbf{M}$. In particular, we have $D_1(\mathbf{M}) = \mathrm{Tr}(\mathbf{M})$ and $D_n(\mathbf{M}) = \det(\mathbf{M})$. Let λ_i denote the ith ordered eigenvalue of $\mathbf{H}\mathbf{H}^\dagger$ and α_i denote the exponential order of λ_i, i.e., $\lambda_i \doteq \mathsf{SNR}^{-\alpha_i}$ with $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_q$. Then, we have

$$D_i \doteq \mathsf{SNR}^{-\sum_{k=q-i+1}^{q}\alpha_k} \quad \text{for } i = 1\ldots q$$

since $\sum_{k=q-i+1}^{q}\alpha_k$ is the smallest among all the combinations of i different α's. Now, we are ready to write

$$\begin{aligned}
\mathcal{O}(r) &\doteq \left\{\mathbf{H} \;:\; 1+\sum_{i=1}^{q}\mathsf{SNR}^i D_i(\mathbf{H}\mathbf{H}^\dagger) < \mathsf{SNR}^r\right\} \\
&\doteq \left\{\mathbf{H} \;:\; \mathsf{SNR}^i D_i(\mathbf{H}\mathbf{H}^\dagger) \dot{\leq} \mathsf{SNR}^r, \quad \forall i = 1\ldots q\right\} \\
&\doteq \left\{\alpha \;:\; i - \left(\sum_{k=q-i+1}^{q}\alpha_k\right) \leq r, \quad \forall i = 1\ldots q\right\} \\
&= \left\{\alpha \;:\; \sum_{k=j+1}^{q}\alpha_k \geq (n-j) - r, \quad \forall j = 0\ldots q-1\right\}
\end{aligned} \tag{50}$$

B. Error event of a rate-n NVD code

Let us now consider the error event of a rate-n NVD code $\mathcal{X}$. We will follow the footsteps of [11]. Using the sphere bound, the error event of ML decoding conditioned on a channel realization $\mathbf{H}$ is

$$\begin{aligned}
\mathcal{E}_{\mathbf{H}} &\subseteq \left\{\mathbf{W} \;:\; \|\mathbf{W}\|_{\mathrm{F}}^2 > \frac{d_{\min}^2}{4}\right\} \\
&\doteq \left\{w \;:\; -w \geq \eta\right\}
\end{aligned}$$

[8]To see this, consider the identity $\det(\mathbf{M}-x\mathbf{I}) = (-1)^n \prod_{i=1}^{n}(x-\lambda_i)$ where λ_i's is the eigenvalues of $\mathbf{M}$.

where $d_{\min}^2$ is the minimum Euclidean distance between two codewords in $\mathcal{X}$, w is the exponential order of $\|\mathbf{W}\|_{\mathrm{F}}^2$ ($\sim \chi^2_{2n_R n_T}$) and η is that of $1/d_{\min}^2$. Therefore, the error probability conditioned on $\mathbf{H}$ is

$$P_{\mathcal{E}_{\mathbf{H}}} \dot{\leq} \text{Prob}\{-w \geq \eta\} \doteq \mathsf{SNR}^{-d_{\mathcal{E}|\mathbf{H}}}$$

where by lemma 4, we have

$$d_{\mathcal{E}|\mathbf{H}} = \begin{cases} \inf\limits_{w \in \mathbb{R}^+} n_R n_T w = 0, & \text{for } \eta \leq 0 \\ \infty, & \text{for } \eta > 0. \end{cases} \tag{51}$$

Then the average error probability becomes

$$\begin{aligned} P_{\mathcal{E}} &= \int P_{\mathcal{E}_{\mathbf{H}}} p_{\mathbf{H}}(\mathbf{H}) \mathrm{d}\mathbf{H} \\ &\dot{\leq} \int_{\eta \leq 0} p_{\mathbf{H}}(\mathbf{H}) \mathrm{d}\mathbf{H} \\ &= \text{Prob}\{\eta \leq 0\}. \end{aligned}$$

Therefore, we get the error event in the high SNR regime

$$\mathcal{E} \,\dot{\subseteq}\, \{\alpha \;:\; \eta \leq 0\} \subseteq \bigcap_{\xi \leq \eta} \{\alpha \;:\; \xi \leq 0\} \tag{52}$$

with ξ being any lower bound on η. Using the same arguments as in [11], with a rate-n NVD code, we can get q lower bounds on $d_{\min}^2$

$$\mathsf{SNR}^{\eta} \doteq d_{\min}^2(\boldsymbol{\alpha}) \,\dot{\geq}\, \mathsf{SNR}^{\delta_j(\boldsymbol{\alpha})}, \quad j = 0, 1, \ldots, q-1$$

with

$$\delta_j(\boldsymbol{\alpha}) = 1 - \frac{q}{n}\frac{r}{j+1} - \sum_{i=q-j}^{q} \frac{\alpha_i}{j+1} \tag{53}$$

Finally, from (52) and (53), we get

$$\begin{aligned} \mathcal{E} &\,\dot{\subseteq}\, \left\{\boldsymbol{\alpha} \;:\; \delta_j \leq 0, \quad \forall j = 0, 1, \ldots, q-1\right\} \\ &= \left\{ \sum_{k=j+1}^{q} \alpha_k \geq (n-j) - \frac{q}{n} r, \quad \forall j = 0 \ldots q-1 \right\} \\ &\doteq \mathcal{O}\left(\frac{q}{n} r\right) \end{aligned}$$

which implies that

$$d_{\mathcal{X}}(r) \geq d_{\text{out}}\left(\frac{q}{n} r\right).$$

APPENDIX VI
PROOF OF THEOREM 4

Consider the channel $\Lambda = \text{diag}(\tilde{\mathbf{H}}_1, \ldots, \tilde{\mathbf{H}}_N)$ with $\tilde{\mathbf{H}}_i$ being similarly defined as $\tilde{\mathbf{H}}$ in (8) except that $\mathbf{G}, \mathbf{H}, \mathbf{B}$ in (3) are replaced by $\mathbf{G}_i, \mathbf{H}_i, \mathbf{B}_i$, respectively. Since one channel use of Λ is equivalent to $2N$ channel uses of a N-relay AF channel, i.e.,

$$C_\Lambda = 2NC_{\mathcal{AF},N} \tag{54}$$

where C_Λ and $C_{\mathcal{AF},N}$ are the capacities of the channel Λ and the equivalent N-relay AF channel, measured by bits per channel use. Therefore, we have

$$d_{\mathcal{AF},N}(r) = d^{\text{out}}_{\mathcal{AF},N}(r) = d^{\text{out}}_{\Lambda}(2Nr) \tag{55}$$

where the first equality comes from the fact that the outage upper bound of the tradeoff can be achieved [6] and the second comes from (54) and the definition of outage since

$$P^{\Lambda}_{\text{out}}(2Nr) = \text{Prob}\{C_\Lambda < 2Nr \log \mathsf{SNR}\} = \text{Prob}\{C_{\mathcal{AF},N} < r \log \mathsf{SNR}\} = P^{\mathcal{AF},N}_{\text{out}}(r).$$

On the other hand, by using a code $\mathcal{C}$ defined above, an equivalent channel model of the N-relay channel is

$$\mathbf{Y} = \sqrt{\mathsf{SNR}}\Lambda\mathbf{X} + \mathbf{Z}$$

with $\mathbf{X} \in \mathcal{X}$. By theorem 3, we have

$$d_{\mathcal{C}}(r) = d_{\mathcal{X}}(2r) \geq d^{\text{out}}_{\mathbf{H}}(2Nr). \tag{56}$$

From (55) and (56), we obtain

$$d_{\mathcal{C}}(r) \geq d_{\mathcal{AF},N}(r).$$

APPENDIX VII
ζ_8 IS NOT A NORM IN $\mathbb{K}$

We prove, in this appendix, that ζ_8 is not a norm of an element of $\mathbb{K} = \mathbb{Q}\left(\zeta_8, \sqrt{5}\right)$. Assume that ζ_8 is a norm in $\mathbb{K}$. That means

$$\exists x \in \mathbb{K}, N_{\mathbb{K}/\mathbb{Q}(\zeta_8)}(x) = \zeta_8. \tag{57}$$

Consider now the extensions described in figure 6. From eq. (57), by considering the left

$$\begin{tikzcd}
 & \mathbb{K} = \mathbb{Q}\left(\zeta_8, \sqrt{5}\right) & \\
\mathbb{Q}\left(\zeta_8\right) \arrow[ur] & & \mathbb{Q}(i, \sqrt{5}) \arrow[ul] \\
 & \mathbb{Q}(i) \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Fig. 6. Two ways of extending $\mathbb{Q}(i)$ up to $\mathbb{K}$

extension of figure 6, we deduce that

$$N_{\mathbb{K}/\mathbb{Q}(i)}(x) = N_{\mathbb{Q}(\zeta_8)/\mathbb{Q}(i)}\left(N_{\mathbb{K}/\mathbb{Q}(\zeta_8)}(x)\right) = \zeta_8 \cdot \tau\left(\zeta_8\right) = -i. \tag{58}$$

Now, we deduce, from the right extension of figure 6 that

$$N_{\mathbb{K}/\mathbb{Q}(i)}(x) = N_{\mathbb{Q}\left(i,\sqrt{5}\right)/\mathbb{Q}(i)}\left(N_{\mathbb{K}/\mathbb{Q}\left(i,\sqrt{5}\right)}(x)\right) = -i. \tag{59}$$

Denote $y = N_{\mathbb{K}/\mathbb{Q}\left(i,\sqrt{5}\right)}(x) \in \mathbb{Q}\left(i, \sqrt{5}\right)$. Then the number

$$z = \frac{1+\sqrt{5}}{2} \cdot y$$

has an algebraic norm equal to i, and belongs to $\mathbb{Q}\left(i, \sqrt{5}\right)$. In [12], it has been proved that i was not a norm in $\mathbb{Q}\left(i, \sqrt{5}\right)$. So, ζ_8 is not a norm in $\mathbb{K}$.

APPENDIX VIII
ζ_{16} IS NOT A NORM IN $\mathbb{Q}\left(\zeta_{16}, \sqrt{5}\right)$

The proof is similar to the one of appendix VII. First, we assume that ζ_{16} is a norm in $\mathbb{K}$. That means

$$\exists x \in \mathbb{K}, N_{\mathbb{K}/\mathbb{Q}(\zeta_{16})}(x) = \zeta_{16}. \tag{60}$$

We deduce that

$$N_{\mathbb{K}/\mathbb{Q}(i)}(x) = N_{\mathbb{Q}(\zeta_{16})/\mathbb{Q}(i)}\left(N_{\mathbb{K}/\mathbb{Q}(\zeta_{16})}(x)\right) = \zeta_{16} \cdot \tau\left(\zeta_{16}\right) \cdot \tau^2\left(\zeta_{16}\right) \cdot \tau^3\left(\zeta_{16}\right) = -i. \tag{61}$$

But we also have,

$$N_{\mathbb{K}/\mathbb{Q}(i)}(x) = N_{\mathbb{Q}(i,\sqrt{5})/\mathbb{Q}(i)}\left(N_{\mathbb{K}/\mathbb{Q}(i,\sqrt{5})}(x)\right) = -i. \tag{62}$$

Denote $y = N_{\mathbb{K}/\mathbb{Q}(i,\sqrt{5})}(x) \in \mathbb{Q}\left(i, \sqrt{5}\right)$. Then the number

$$z = \frac{1+\sqrt{5}}{2} \cdot y$$

has an algebraic norm equal to i, and belongs to $\mathbb{Q}\left(i, \sqrt{5}\right)$ which is a contradiction.

REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—Part I: system description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.

[2] ——, "User cooperation diversity—Part II: implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, pp. 1939–1948, Nov. 2003.

[3] J. Laneman and G. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[4] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[5] R. Nabar, H. Bölcskei, and F. Kneubühler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Select. Areas Commun.*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.

[6] K. Azarian, H. El Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inform. Theory*, Apr. 2005, submitted for publication.

[7] T. Hunter and A. Nosratinia, "Coded cooperation in multi-user wireless network," *IEEE Trans. Wireless Commun.*, 2005, submitted for publication.

[8] T. Hunter, S. Sanayei, and A. Nosratinia, "Outage analysis of coded cooperation," *IEEE Trans. Inform. Theory*, 2005, submitted for publication.

[9] P. Mitran, H. Ochiai, and V. Tarokh, "Space-time diversity enhancements using collaborative communications," *IEEE Trans. Inform. Theory*, vol. 51, pp. 2041–2057, June 2005.

[10] L. Zheng and D. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[11] P. Elia, K. Kumar, S. Pawar, P. Kumar, and H. Lu, "Explicit, minimum-delay space-time codes achieving the diversity-multiplexing gain tradeoff," *IEEE Trans. Inform. Theory*, Sept. 2004, submitted for publication. [Online]. Available: http://ece.iisc.ernet.in/~vijay/csdpapers/explicit.pdf

[12] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The Golden code: a 2 × 2 full-rate space-time code with nonvanishing determinants," *IEEE Trans. Inform. Theory*, vol. 51, no. 4, pp. 1432–1436, Apr. 2005.

[13] F. Oggier, G. Rekaya, J.-C. Belfiore, and E. Viterbo, "Perfect Space Time Block Codes," submitted to *IEEE Trans. on Inf. Theory*, 2004, http://www.enst.fr/~belfiore/publi.html.

[14] S. Tavildar and P. Viswanath, "Approximately universal codes over slow fading channels," February 2005, submitted to *IEEE Trans. on Inform. Theory*.

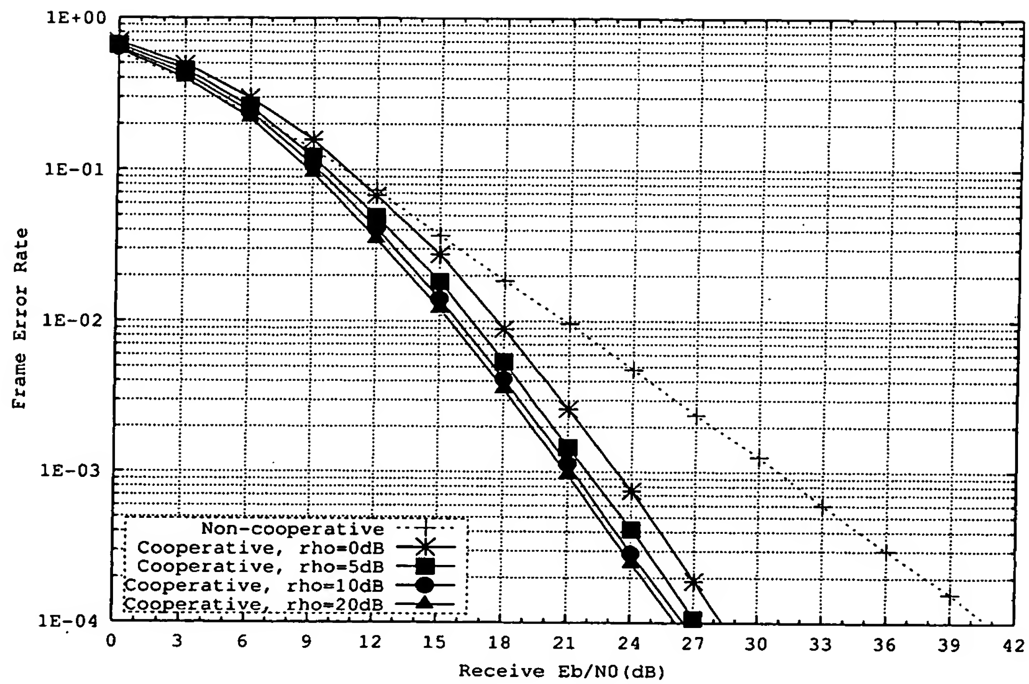
[15] J.-C. Belfiore and G. Rekaya, "Quaternionic lattices for space-time coding," in *Proceedings of the Information Theory Workshop.* IEEE, Paris 31 March - 4 April 2003 ITW 2003.

[16] A. A. Albert, "Structure of Algebras," AMS Colloqium Pub. XXIV, 1961, AMS, Providence.

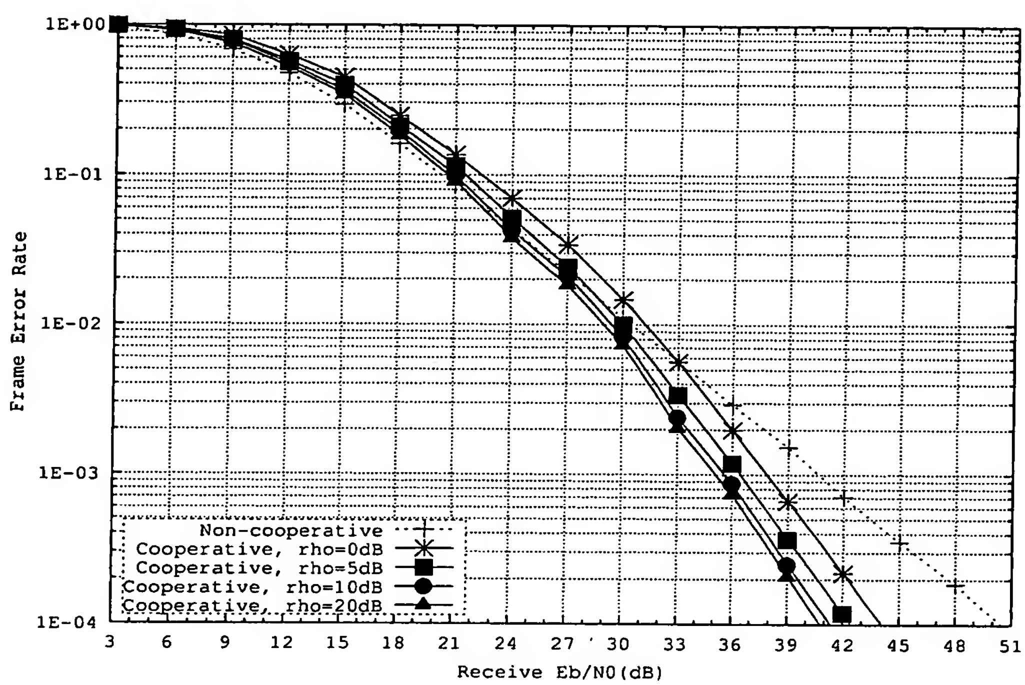
[17] T. Rappaport, *Wireless communications: principles and practice.* New Jersey: Prentice Hall, 1996.

[18] T. Ratnarajah, R. Vaillancourt, and M. Alvo, "Complex random matrices and Rayleigh channel capacity," *Communications in Information and Systems*, vol. 3, no. 2, pp. 119–138, oct 2003.

[19] A. Tulino and S. Verdu, "Random matrix theory and wireless communications," *Foundations and Trends in Communications and Information Theory*, vol. 1, no. 1, pp. 1–182, 2004.

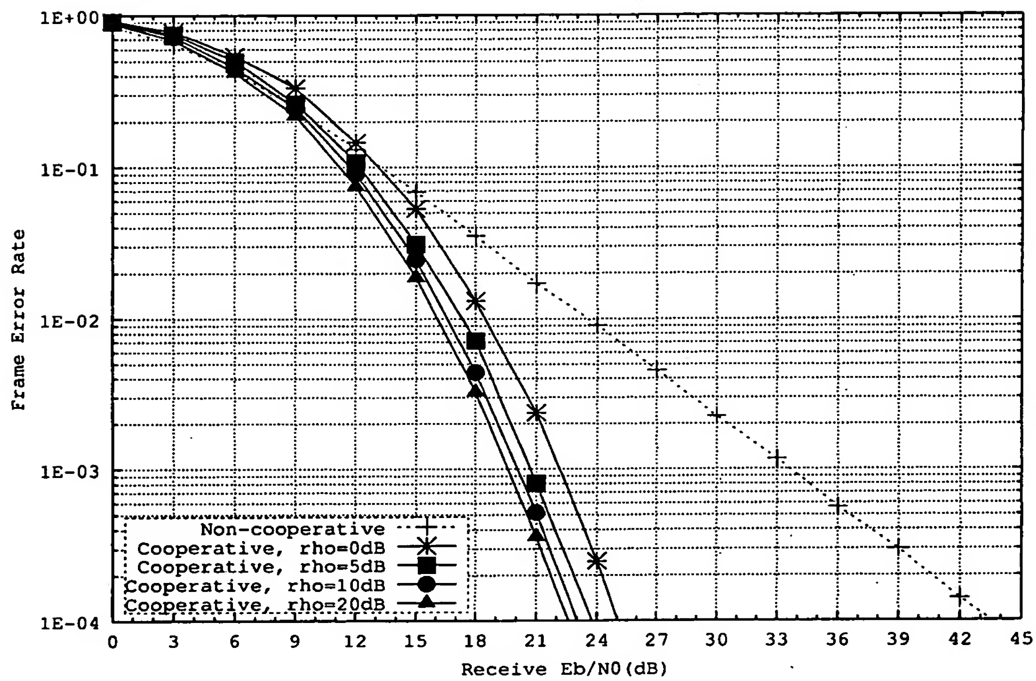


(a) 4-QAM

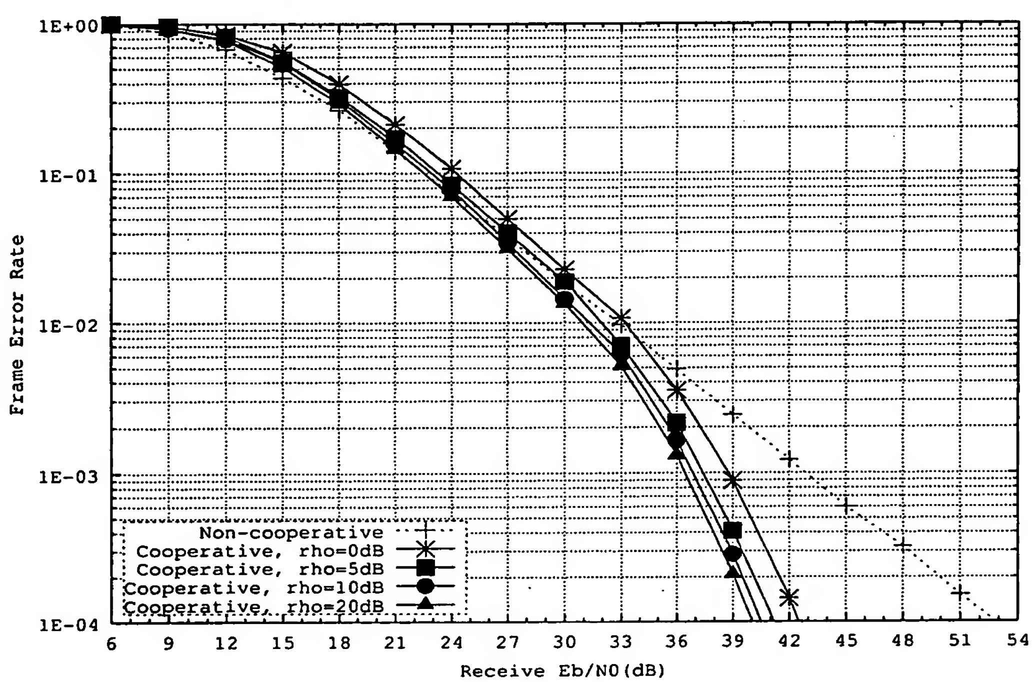


(b) 64-QAM

Fig. 7. AF single-antenna single-relay channel, Rayleigh fading, Golden code



(a) 4-QAM



(b) 64-QAM

Fig. 8. AF single-antenna 4-relay channel, Rayleigh fading, $\mathcal{C}_{4,1}$

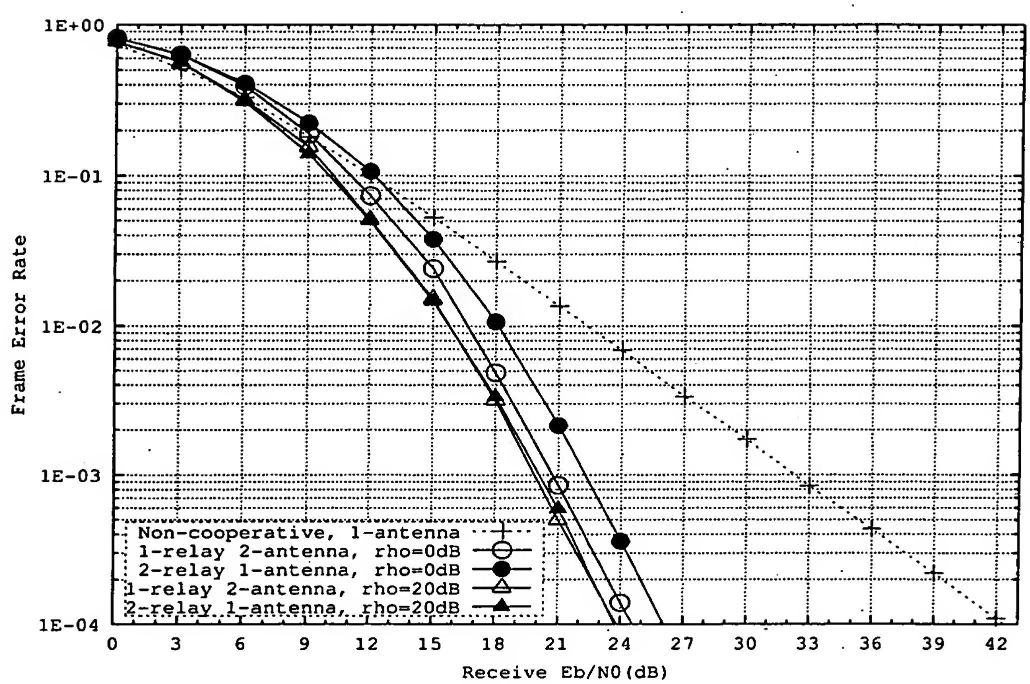
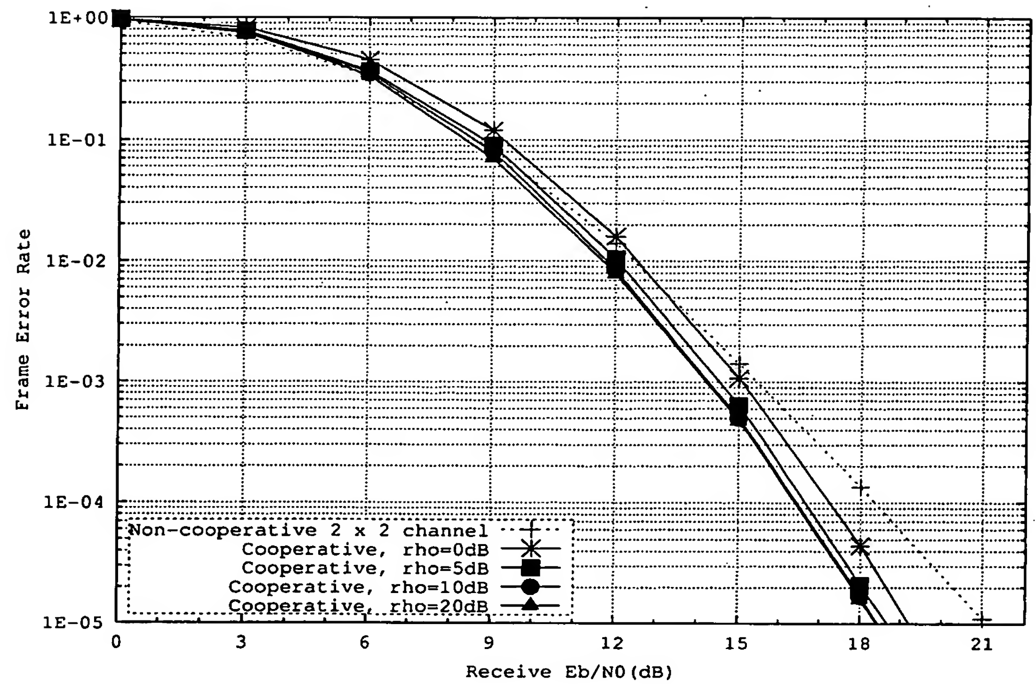
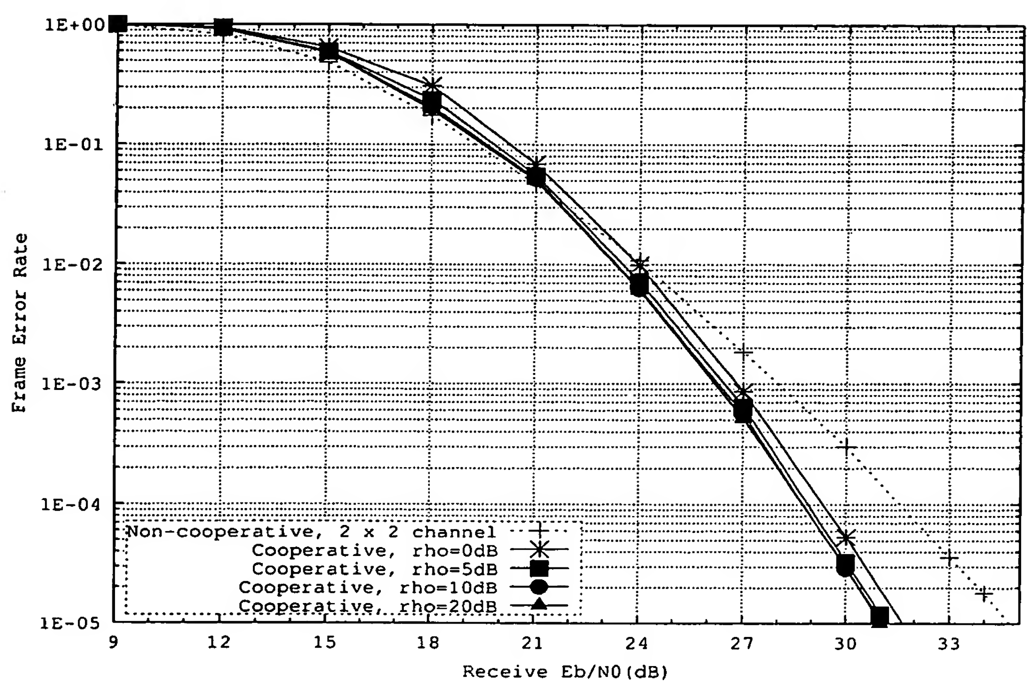


Fig. 9. AF single-relay $(1, 2, 1)$ channel *vs* . single-antenna 2-relay channel, Rayleigh fading, $C_{2,1}$ with 4-QAM



(a) 4-QAM



(b) 64-QAM

Fig. 10. AF single-relay $(2, 2, 2)$ Rayleigh channel, 4×4 Perfect code

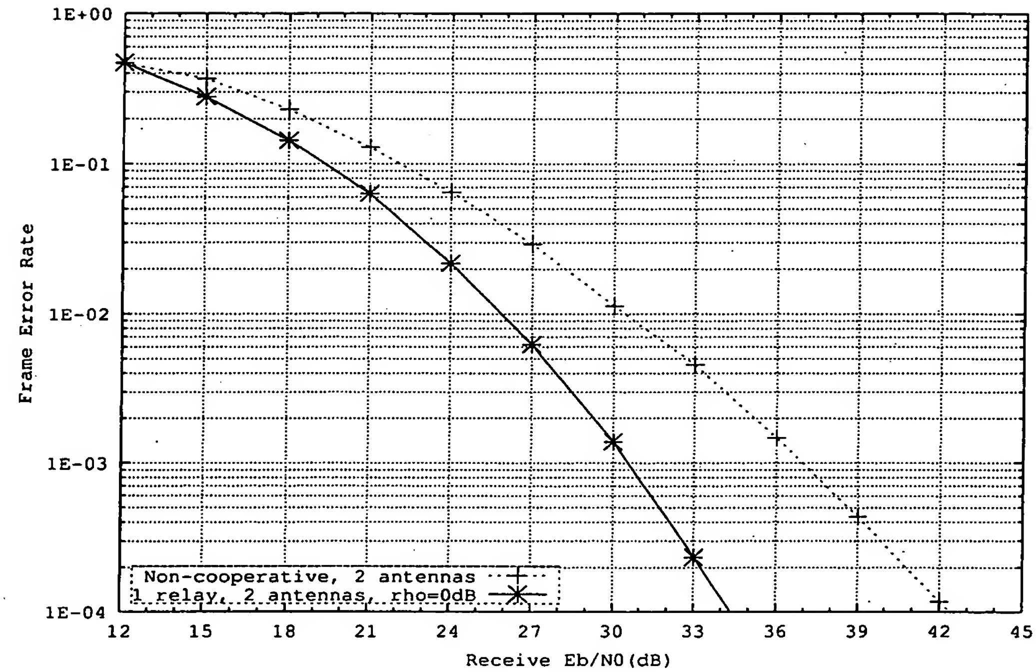


Fig. 11. AF single-relay (2, 2, 2) Rayleigh channel, log-normal shadowing with variance 7dB, 4-QAM, 4 × 4 Perfect code